5 Effects of prestressing on concrete elements and structures, design of prestressing

Prestressed concrete structures are, more than any other type of civil engineering structures, characterised by the variety of construction and service stages with changing (i) loading condition, (ii) prestressing or (iii) structural scheme of the structure. The simplest situation represents three stages: (i) during stressing, (ii) at the moment when all loads start to act and (iii) at the end of structure lifespan when all the loads are active and long-term prestressing losses have taken place to the full extent. In prefabricated structures, we must add the phase of storage or transport of the prestressed element. Before approaching the very analysis of the prestressing effects, we will, in the following chapter, try to provide a list of important ***production and service stages in which the effects of prestressing must be analysed*** in common prestressed structures. The analysis of more complex prestressed structures built in stages will be presented in chapter 7.

5.1 Action stages of prestressed structure

The initial phase of action of the structure includes stages prior to, during, and after the introduction of prestressing. In the period ***before the prestressing is introduced*** (before transfer), the capability of the concrete to resist tension is only small. Therefore, proper curing must be provided to prevent its drying (this also means shrinkage) and eliminate sudden temperature changes. Any cracks could completely eliminate the capability of the concrete to transfer tensile stress. Under certain circumstances the cracks would not necessarily close after prestressing, and even though they would not have to be a factor limiting the load bearing capacity or serviceability of the structure, they could give a reason for objections of the client.

During stressing (time t_0, t_{g0}) the prestressing reinforcement is subjected to maximum stress. The largest stress is under the anchor and friction losses reduce its size along the length of the tendon. Exceptionally, individual wires or the whole strand may get ruptured. If the prestressing reinforcement gets broken repeatedly, its physical and mechanical properties must be verified and the stressing device must be tested. Any reduction of prestressing forces must be consulted with the responsible structural engineer. The stressing is a critical operation also in terms of anchors and concrete in the anchorage zone, as the strength at the time of prestressing usually has not yet reached the 28-day strength. Therefore, the prestressing must be introduced successively and symmetrically.

As the element lifts up from the rest during stressing (cambering effect of prestressing), its self-weight g_0 is activated and it acts against this cambering. In order to utilise the self-weight of the element to reduce the effects of prestressing, required boundary conditions must be provided, in particular adequately rigid base with suitable pads in the places where the element is supported. If the ends of the cambering element sink into the base, the self-weight would be balanced by the contact stress, it would not act against the prestressing and the top fibre at midspan could get damaged due to tension.

During the period ***after the prestressing was introduced*** (time t_a, t_{g0}), the level of prestressing is still relatively high as it is only reduced by immediate losses. Apart from the self-weight and prestressing, usually no other load has been introduced yet. However, the supports often change (i) during the manipulation and transport of the prefabricated member or (ii) on the removal of the formwork or (iii) on the removal of temporary supports. These factors often subject the structure to dynamic loading. It is necessary to pay attention to proper supporting of the member during any manipulation and during storage. For example, if the manipulation hanger is located close to the middle of the span of the future simply supported beam, it causes cracks at the top fibre at midspan and damage of the member.

During the construction, superimposed dead load is usually introduced (time t_{g1}), sometimes the structure is re-stressed and the boundary conditions change. These operations may result in conditions that are more critical for the structure, in terms of its load bearing capacity or crack formation, than the full-operation conditions after the completion of the structure. Therefore, the behaviour of the structure in all construction stages must be verified. Chronological succession of the (i) changes in the structural scheme, (ii) application of loads and (iii) re-stressing of the structure must be strictly followed during the construction.

To give a simple example of an incorrect procedure for the application of superimposed dead loads, we may use a simple beam with a cantilever at one of its ends as in Fig. 5-1. If the prestressing of the beam was designed only to resist the load acting along the whole length of the beam, then the successive application of the load starting from the right-hand-side-end results in the bending moment identical to the bending moment on a simply supported beam, i.e. the condition the structure has not been designed for. Consequently, the formation of cracks takes place at the bottom fibre in the vicinity of the interior support of the beam.

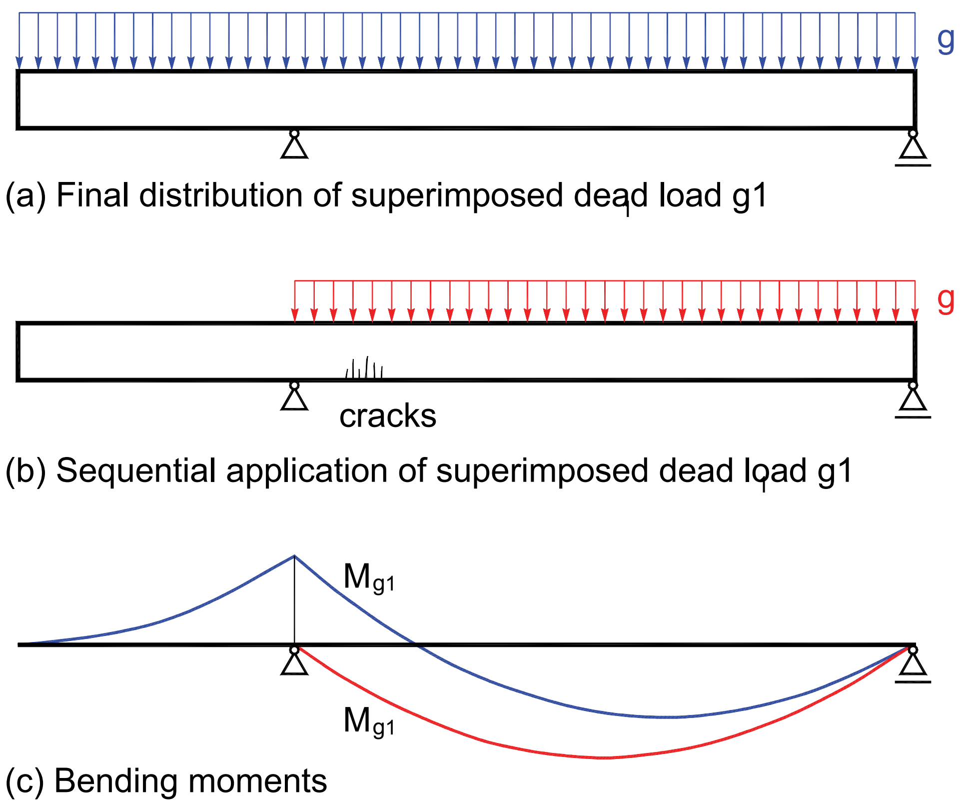

Fig. 5-1 Cracks due to an incorrect sequence in the application of superimposed dead load

In the service stages (time t_q, t_∞), also variable loads appear in addition to permanent loads and prestressing. Similarly to other types of civil engineering structures, the ***ability of the structure to resist the "ultimate" load*** must be verified. The ultimate limit state serves the purpose. Most structures, however, are not subjected to such loading during the whole lifespan that the ultimate limit state is reached. On the other hand, the structure will be very often subjected to common loading conditions, and therefore, it is extremely important to ***verify the behaviour of the structure subjected to "service" loads***. Some up-to-date standards introduce a limit state termed stress-limitation (CEB-FIP 1990, EN 1992-1-1), some other standards more precisely specify what is termed permissible (allowable) stresses (ACI 318M-05, ČSN 73 6207). During a normal operation, the stress should not exceed the permissible stress. (i) As it is the service loads the structure should be able to resist during most of the lifespan and (ii) in order to simplify the calculation, the permissible stress criteria are often used for the design of the structure. What is also important in a prestressed structure, is the ***analysis of the effects of long-term loads***. The ratio of the (i) prestressing level and (ii) intensity of other long-term loads determines whether the structure deflects or cambers and to what extent is this deformation raised by the creep of concrete (it results from the long-term load). In some situations also the level of ***load immediately prior to cracking*** is frequently determined. Cracks are what dramatically reduces the stiffness of load bearing elements. The level of load immediately prior to cracking may therefore show the limit (i) when the deflection starts to increase in fast to excessive manner, or (ii) when the fatigue strength falls down. The load immediately prior to cracking is, of course, very important for water-tight structures and structures exposed to an aggressive environment.

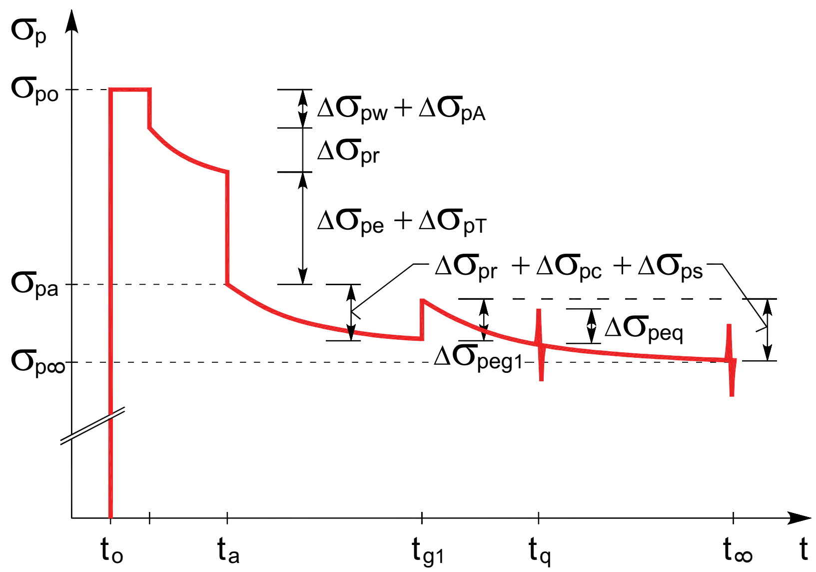

Fig. 5-2 Changes in prestressing in pre-tensioned prestressed element

In each of the above-mentioned construction and service stage of the prestressed structure, it is essential to reliably ***determine the size of the currently acting prestressing force***. The calculation of prestressing losses (changes) serves this purpose, see Fig. 5-2. Only the knowledge of the precise value of the acting prestressing force makes it possible to (i) evaluate the ***effects of prestressing*** at a given time instant (given stage), (ii) superpose them with the effects of external loads and (iii) assess them. Up-to-date calculation methods allow for the calculation of changes in the prestressing (in particular long-term ones) directly in the calculation model for the structural analysis of the effects of external loads. Thus it is possible to take into account more precisely e.g. the (i) static action of tendons during stressing, (ii) location of the tendons in the calculation of changes in prestressing due to the instantaneous elastic strain of the structure caused by a variable load, (iii) interaction of creep and shrinkage of the concrete and relaxation of the reinforcement, etc., see chapter 7.6.

5.2 *Equivalent load method*

If we know the distribution of the prestressing force along the length of the tendon, we can examine the effect of the tendon on the structure. In the past, various methods were used to determine the effects of prestressing mainly due to the difficulties connected with the static calculation of general effects of prestressing on statically indeterminate structures. With the present capabilities of computers and universality of the finite element method, the static solution of the effects of prestressing is no longer a critical issue and we will not address it in this book. The only remaining problem is thus the determination of the action of the force in the tendon, i.e. the determination of the equivalent load due to prestressing.

If we apply the equivalent load on a given structure, we obtain the distribution of internal forces due to prestressing. Let us note that it is just the action of the force in the prestressing tendon that makes it possible to ***actively change the distribution of internal forces*** over the structure, which distinguishes the prestressed concrete from all other civil engineering structures that passively resist the external load, see chapter 1.1.

5.2.1 Force action of a tendon on the concrete

The concrete beam is subjected to the effects of tendon (i) through a concentrated force at anchoring, and (ii) through the forces in points where the direction of the tendon changes. To explain this, we will use a model example of a post-tensioned prestressed concrete. It is clear from Fig. 5-3 (a) that the concrete in the location of the anchor is at the analysed instant subjected to prestressing force P_A that, due to the friction between the tendon and tendon ducts, changes over the length of the tendon. In the analysed point its size is P. Let us assume that the direction of the tendon changes at this point. This, inevitably, deviates also force P and, consequently, the force acts under a different angle at the right-hand side and left-hand side of the point. The resultant of the prestressing forces is then force R that acts on the concrete. The concrete applies a force of the same size against the tendon, which means that equilibrium is satisfied.

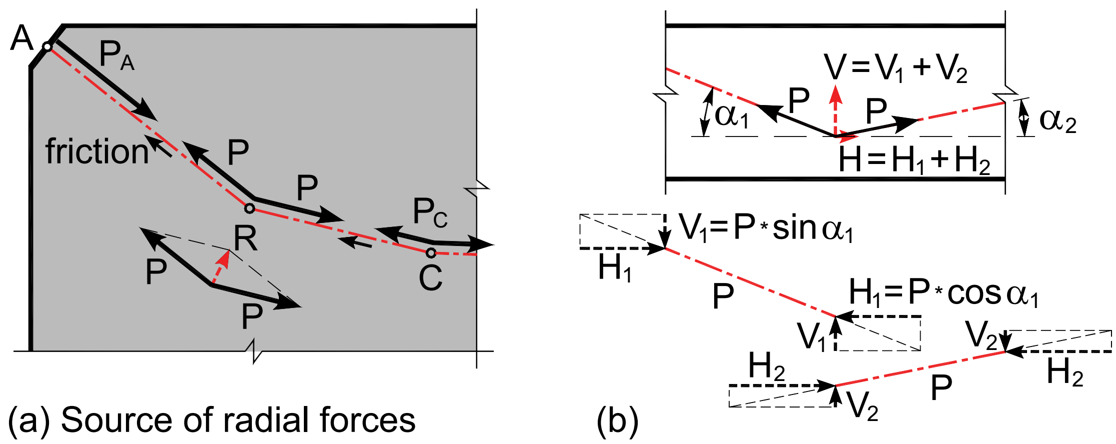

Fig. 5-3 Forces caused by prestressing acting on concrete

The action of the force in a tendon on the concrete at the analysed point can be expressed by means of horizontal force H and vertical force V that can be obtained as the sum of horizontal and vertical components of the prestressing force acting at the right-hand side and left-hand side of the point respectively - see Fig. 5-3 (b). Forces H and V can also be calculated as the components of force *R*. It is thus clear that the forces are equivalent to the effects of prestressing at the given point of the tendon. The system of all forces resulting from prestressing and acting on the concrete is termed ***equivalent load***. It is self-evident that if we move (shift) the force from the actual action-point, we must modify the system of forces accordingly and add a corresponding moment.

If an automated calculation is applied, the equivalent load is usually determined by means of a suitable program without any simplifying assumptions. Fig. 5-4 (a) for example shows the action of force in a simple parabolic tendon. Due to friction, force P_A decreases from the value at the stressed end down to value P_B which also reduces, along the length of the tendon, radial forces *r* that occur due to the change in tendon direction. It is obvious that the system of forces introduced by the tendon is always in the state of equilibrium – equivalent load creates a self-equilibrating system.

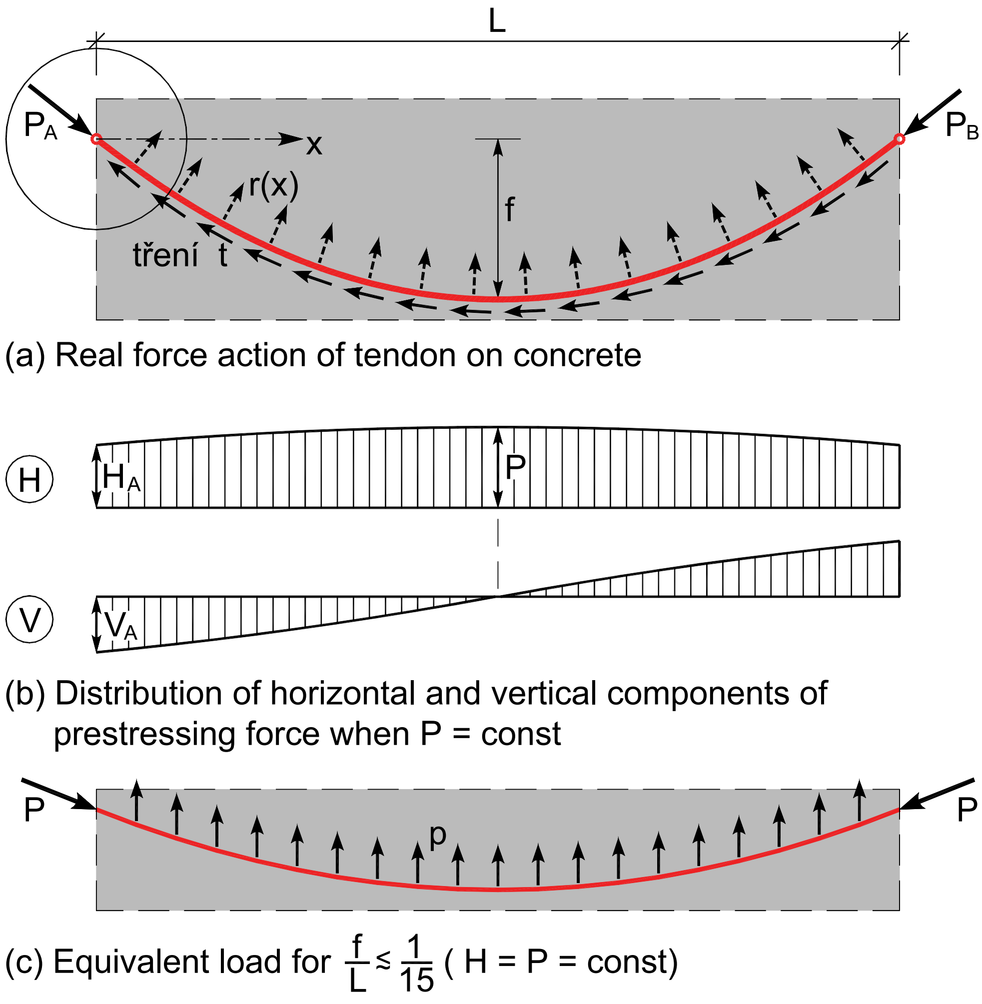

Fig. 5-4 Equivalent load for a parabolic tendon

For manual calculation of the equivalent load, however, we usually accept the simplifying assumptions. Quite often the average value of the prestressing force is considered constant along the length of the tendon. The distribution of horizontal force *H* and vertical force *V* then corresponds to cosine and sine of the angle between (i) the tangent to the tendon and (ii) the horizontal plane, see Fig. 5-4 (b). It is obvious that the angular change along the length of the tendon is rather small for commonly used tendons and, therefore, horizontal force H is practically constant. Generally, this simplifying assumption can be accepted for parabolic tendons with the ratio of camber f to length of parabola L lower than 1/15, i.e. for "shallow" tendons. If these assumptions are met, uniformly distributed vertical load p can be considered as the equivalent load instead of radial forces r – see Fig. 5-4 (c).

5.2.2 Equivalent load for a parabolic tendon profile

The size of the equivalent load p for a parabolic tendon will be calculated using the example of a simply supported beam with a parabolic tendon as in Fig. 5-5. Let us assume again that $P=H=$ *const.* and $e_{pA} = e_{pB}$. The moment effect of the tendon in given section x with regard to the centroid of the cross-section can be, in statically determined structures, obtained as

$$M_p(x) = H e_p(x)\,, \qquad (5.1)$$

where $e_p(x)$ is the eccentricity of the tendon related to the centroidal line of the beam. The axial force due to prestressing is considered negative as it produces compression. The eccentricity can be expressed from the parabola equation as

$$e_p(x) = -\frac{4f}{L^2}x^2 + \frac{4f}{L}x + e_{pA}\,. \qquad (5.2)$$

Substituting (5.2) into (5.1) and using Schwedler theorem, it follows

$$p = -\frac{d^2 M_p(x)}{dx^2} = -H\frac{d^2 e_p(x)}{dx^2} = H\frac{8f}{L^2}\,. \qquad (5.3)$$

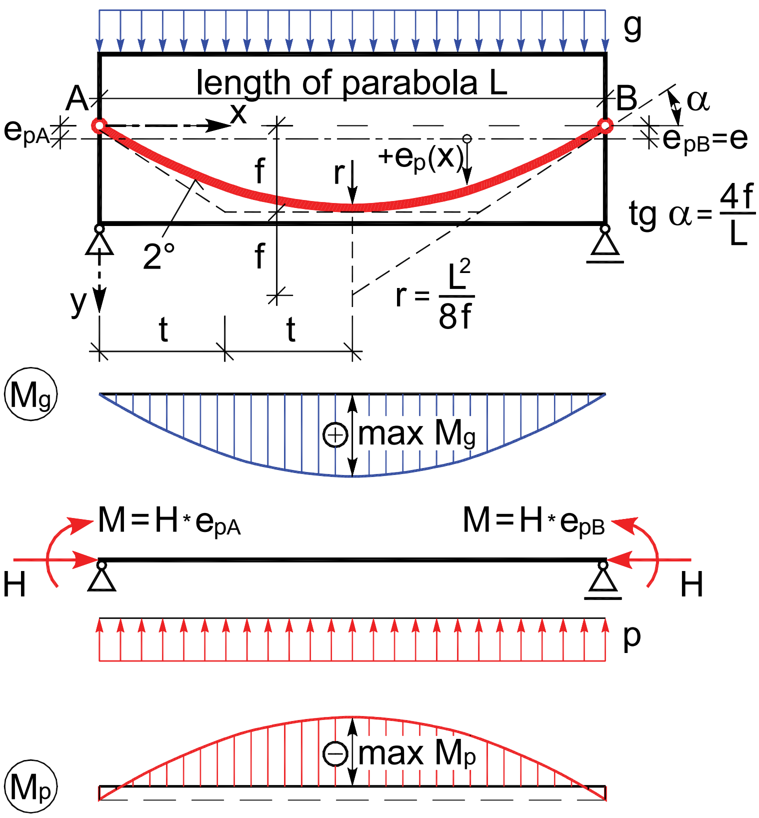

Fig. 5-5 Beam with a parabolic tendon

In addition to distributed load p, the equivalent load of the analysed beam is formed by concentrated forces H and V in the place where the tendon is anchored. The forces act on eccentricities $e_{pA} = e_{pB}$, and thus if we shift them to the centroid of the beam cross-section, the equivalent load must be complemented by the appropriate concentrated moment M.

If we apply the equivalent load to a simply supported beam, we obtain the distribution of internal forces due to prestressing, e.g. bending moment M_p. With regard to the self-equilibrium of the equivalent load, no reactions occur in the simply supported beam due to the action of prestressing.

As already said, also the simple formula (5.1) can be used for the calculation of this moment, without any necessity to apply an “awkward” procedure and determine the equivalent load in advance. Unfortunately, this is applicable only to statically determined structures. On the other hand, the ***equivalent load method is general*** and can be employed with any type of structure, see chapter 5.3.

Moreover, this method makes it possible to monitor the force action of the prestressing tendon against the action of other load types. For example, as it follows from Fig. 5-5, self-weight of the beam g can be balanced by uniformly distributed equivalent load p, so that moment M_g will be eliminated by moment M_p due to prestressing (on condition that $e_{pA} = e_{pB} = 0$). Consequently, the ***equivalent load method better expresses the fact that prestressing can be used to change the distribution of internal forces*** in the structure. The simply supported beam subjected to bending will thus be just exposed to axial compression.

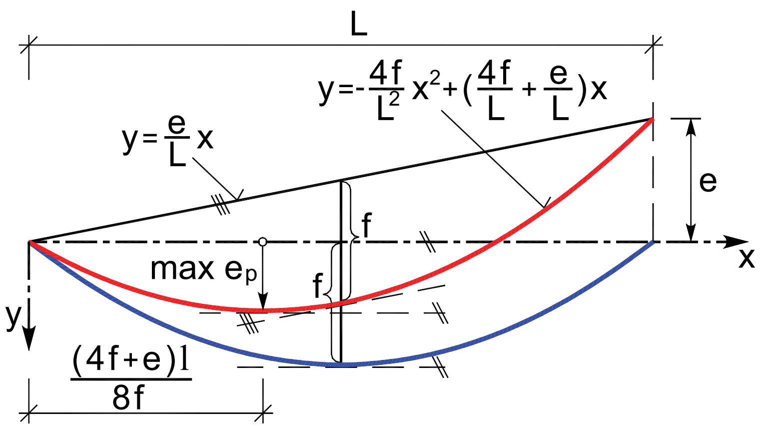

Fig. 5-6 Geometric properties of a parabolic tendon

Without any relation to the example above, let us show how to determine the size of equivalent load p from an unsymmetrical parabolic tendon. In prestressed continuous beams, unsymmetrical parabolic tendons with eccentricity e at the lifted end are often used in end spans, see Fig. 5-6. It follows from the parabola equation that the vertex of parabola (the point with the maximal eccentricity and horizontal tangent) shifts slightly from the centre of length L into the point with coordinate $x=(4f+e)L/(8f)$. Nevertheless, the axis of the parabola remains vertical. On the assumption that $H=const.$, also equivalent load p is a constant uniformly distributed vertical load and can be calculated from formula (5.3).

5.2.3 Beams with variable cross-section

Generally, the calculation of the equivalent load requires that the eccentricity of the tendon be respected. To explain this, we can advantageously use an example of a beam of a variable cross-section with a straight tendon, see Fig. 5-7. When we shift the concentrated forces at anchoring from the actual point of action, we have to complement the equivalent load with the appropriate moment – similarly to the example in Fig. 5-5. In addition, a vertical concentrated force acts in the point where the centroidal line bends. The existence of this force can be easily explained if we draw the eccentricities of the tendon with reference to the straightened centroidal line along the whole length of the beam.